SMT-Based Modeling and Verification of Spiking Neural Network

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Satisfiability Modulo Theory

- In computer science and mathematical logic, **satisfiability modulo theories (SMT)** is the problem of **determining whether a mathematical formula is satisfiable.**

Linear Arithmetic Theories

- In linear arithmetic theories, atoms are of the form:

$$a_1 x_1 + \ldots + a_n x_n \bowtie b$$

where $\bowtie$ is one of: $=, \neq, <, >, \leq, \geq$
All symbols are interpreted with their usual meaning in arithmetic

- Example of atom:

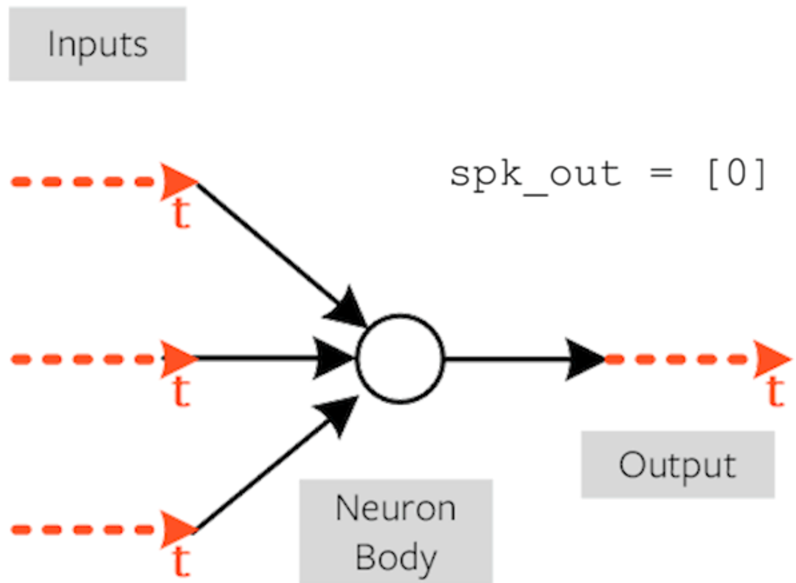
$$x + y + 2z \geq 10$$

- Example of formula:

$$x \geq 0 \wedge (x + y \leq 2 \vee x - y \geq 6) \wedge (x + y \geq 1 \vee x - y \geq 4)$$

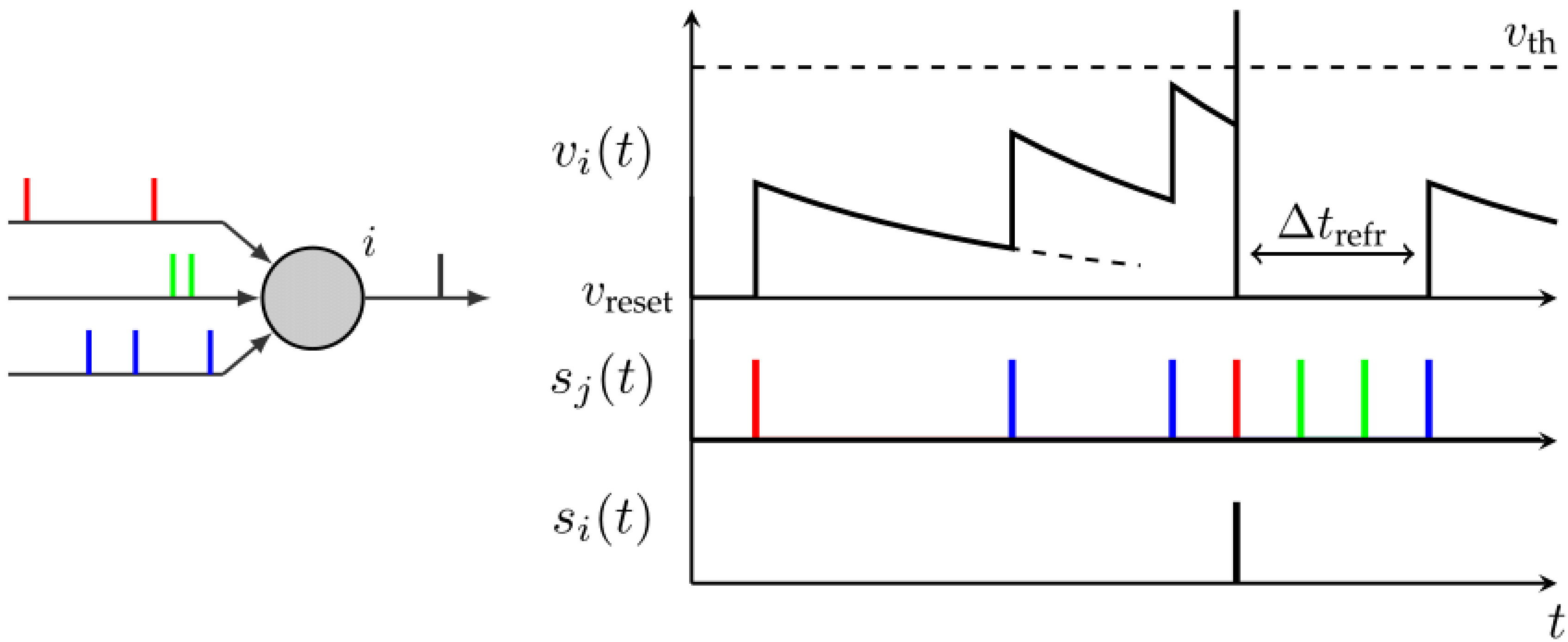
- Variables can be of real sort ($\mathbb{R}$) or integer sort ($\mathbb{Z}$)
- If all vars are $\mathbb{R}$ we have a problem of Linear Real Arithmetic (LRA)
- If all vars are $\mathbb{Z}$ we have a problem of Linear Integer Arithmetic (LIA)

Spiking Neural Network

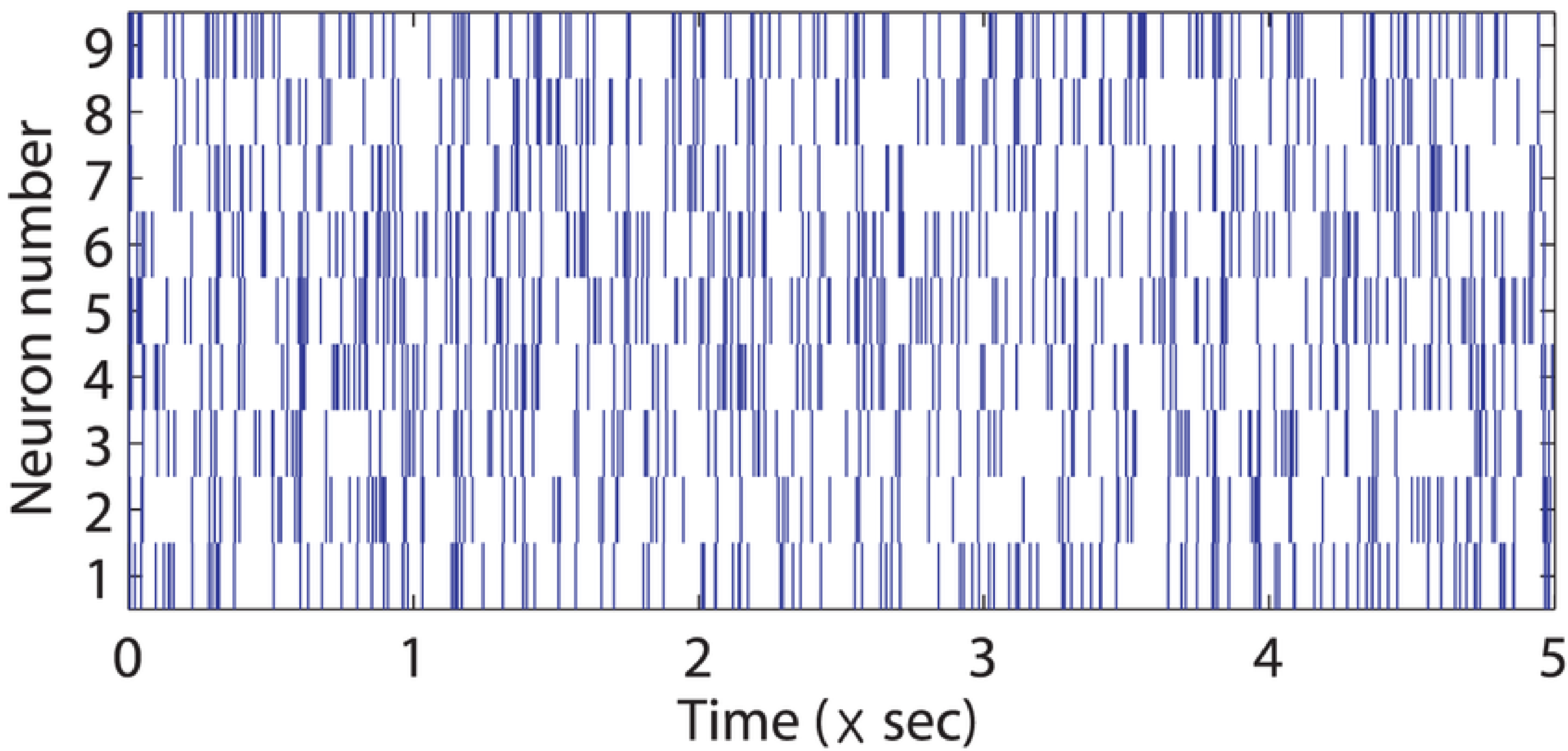


https://github.com/jeshraghian/snntorch

Leaky Integrate and Fire Neuron



Complex Spike Train



First Order Logic

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) \quad \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) \quad \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) \quad \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) \quad \text{associativity of } \vee \\
\neg(\neg\alpha) &\equiv \alpha \quad \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) \quad \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) \quad \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) \quad \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) \quad \text{De Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) \quad \text{De Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) \quad \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) \quad \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

Soundness and Completeness